Relationships between voltage and angle stability of power systems

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This paper discusses modelling and theoretical issues associated with voltage and angle stability of power systems. A time-scale decomposition is performed to illustrate how the critical modes can be identified with reduced-order models and the bifurcation phenomena can be explained with these low order models. Examples are given for single and multimachine systems. Copyright © 1996 Elsevier Science Ltd

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I. Introduction

Power system dynamics are closely related to the mechanical and electrical dynamic state variables of all the synchronous machines interconnected through the network. Historically, power system stability has been associated with the generator rotor angle dynamics. In this paper we re-examine the issue of the classic 'steady state' (non-oscillatory) angle stability in the light of recent research relating to voltage stability problems. We intend to demonstrate that the Saddle Node Bifurcation (SNB) associated with the maximum power delivered by a synchronous machine under constant excitation is not an 'angle' stability problem, but on the contrary it initiates a slow demagnetization process sensed in the network as a voltage decay with negligible frequency error. It is only during this process that the actual loss of synchronism point is met.

Voltage stability is predominantly load stability\(^1,2\). However, it cannot be completely separated from the dynamics of the synchronous generators which provide both the power and the voltage to the load buses. In a traditional analysis, voltage stability was initially considered from a load flow perspective, in which the generators were simply regarded as 'PV buses'. The contradiction of using the term 'stability' to refer to a problem with no dynamics was recognized eventually and it is now generally accepted that 'voltage collapse is ultimately a dynamic phenomenon'\(^1,2\). However, the dynamics involved in voltage stability have in many studies been restricted to load buses only, involving for instance load tap changing (LTC) transformers, restorative loads\(^3,4\), etc., whose time frame is of the order of one or more minutes. Many voltage stability incidents evolve in this time frame, for which the generator dynamics can be legitimately substituted by appropriate equilibrium conditions\(^5,6\).

In many cases, however, a voltage instability scenario originating in the midterm time-scale gradually manifests itself as transient instability, and the equilibrium of generator dynamics is eventually lost\(^7\) leading to a loss of synchronism. Also, there is at least one case reported in the literature\(^8\) as a voltage collapse incident, for which synchronous generators seem to be the key factor responsible for the instability.

The theoretical question sketched above can be stated as follows: Since voltage stability is directly or indirectly linked to the 'steady state' angle stability problem is there a framework for correctly decoupling angle and voltage stability problems? An answer to a similar question is attempted in Reference 9 for oscillatory instability using the concept of an 'incremental reactive current flow network'. In our analysis, the answer is sought by applying singular perturbation analysis to an unregulated multimachine power system in order to decompose it formally into a slow and fast subsystem. The physical concept of time-scale decomposition was introduced in Reference 10, where the power system was decomposed into a mid-term and a transient time-scale. In our approach we proceed further to decompose the generator dynamics (in the transient time-scale) into voltage (flux)
and shaft (electromechanical) dynamics. Using this approach it is clearly demonstrated that there is only one mechanism for generator SNB, which involves both voltage and angle, but not frequency or power.

A formal decomposition of the dynamics of a multi-machine, unregulated system into fast electromechanical oscillations and slow flux-voltage response modes is undertaken in Section II. Section III suggests an approximate method of analysis for a partly regulated system, and Section IV presents three illustrative examples.

II. Decomposition of machine dynamics

II.1 Slow manifold in multimachine systems

A power system consisting of \( m \) synchronous machines can be described in the transient time-scale by the following set of equations, with the machines represented using the one-axis model:\( ^{11} \):

\[
\begin{align*}
\delta &= \omega \\
\dot{\omega} &= \omega - T_{M} \omega = P_{m} - P(\delta, E_{q}') - \omega \omega D \omega \\
T_{d} \ddot{E}_{q}' &= E_{q} - E(\delta, E_{q}')
\end{align*}
\]

where \( \delta, \omega, E_{q}', \) and \( E_{q} \) are \( m \times 1 \) vectors representing the rotor angles, speed deviations, internal voltages (field fluxes), and excitation voltages of the \( m \) machines, respectively, \( T_{M}, D, \) and \( T_{d} \) are \( m \times m \) diagonal matrices containing the mechanical starting times \( (T_{Mi} = 2H_{i}) \), the damping terms, and the field open circuit time constants, respectively, and finally \( \omega_{0} \) is the synchronous speed. The machines considered in this section are without automatic voltage regulators and the mechanical power input \( P_{m} \) is assumed to be constant. The operation without AVR is possible when the machines are under manual voltage control, or when they have reached their excitation limits. The functions \( P \) and \( E \) are \( m \)-valued functions of the state variables \( \delta \) and \( E_{q}' \), depending on the interconnection between machines and loads.

The multimachine model (1)–(3) can be decomposed into two subsystems, a slow one consisting of flux-decay modes, and a fast one describing electromechanical oscillations. To achieve this, the following parameters are introduced:

\[
\begin{align*}
\epsilon &= \sqrt{\frac{2H_{0}}{\omega_{0}}} \\
H_{0} &= \frac{1}{m} \sum_{i=1}^{m} H_{i} \\
\omega &= \epsilon \omega \\
H &= \text{diag}[H_{i}/H_{0}]
\end{align*}
\]

Using the above notation, the system (1)–(3) takes the following standard form for singular perturbation analysis:\( ^{12} \):

\[
\begin{align*}
\epsilon \dot{\delta} &= \omega' \\
\epsilon \omega' &= H^{-1} \left[ P_{m} - P(\delta, E_{q}') - \epsilon \frac{2H_{0}}{2H_{0}} \omega \omega D \omega \right] \\
\ddot{E}_{q}' &= T_{d}^{-1} \left[ E_{q}' - E(\delta, E_{q}') \right]
\end{align*}
\]

In (8)–(10) \( \delta, \omega' \) are the fast variables and \( E_{q}' \) are the slow variables.

Consider the \( m \)-dimensional manifold in the state space of the system (8)–(10) defined by the 2\( m \) equations:

\[
\begin{align*}
\delta &= h_{1}(E_{q}') = h_{0} + \epsilon h_{11} + O(\epsilon^{2}) (11) \\
\omega' &= h_{2}(E_{q}') = h_{20} + \epsilon h_{21} + O(\epsilon^{2}) (12)
\end{align*}
\]

The manifold defined by \( h_{1}, h_{2} \) will be an integral or invariant manifold for the fast shaft dynamics if the following conditions hold, which guarantee that a trajectory starting on the manifold will remain on it for all time:

\[
\begin{align*}
\epsilon \frac{\partial h_{1}}{\partial E_{q}'} T_{d}^{-1} \left[ E_{q}' - E(h_{1}, E_{q}') \right] &= h_{2} \\
\epsilon \frac{\partial h_{2}}{\partial E_{q}'} T_{d}^{-1} \left[ E_{q}' - E(h_{1}, E_{q}') \right] \\
&= H^{-1} \left[ P_{m} - P(h_{1}, E_{q}') - \epsilon \frac{2H_{0}}{2H_{0}} \omega \omega D \omega \right]
\end{align*}
\]

Note that \( h_{1}, h_{2} \) are functions of \( E_{q}' \).

The above conditions (13) and (14) are obtained by substituting (11) and (12) into (8) and (9) and making use of (10). It is not possible in general to solve equations (13) and (14) in order to obtain analytically the integral manifold \( h_{1}, h_{2} \) (also called the slow manifold) of the system. An approximate slow manifold can be found, however, by substituting \( h_{1}, h_{2} \) as a power series in \( \epsilon \), as in (11) and (12) and equating the \( \epsilon^{0} \) and \( \epsilon^{1} \) terms. This process gives the following set of equations:

\[
\begin{align*}
\epsilon \frac{\partial h_{10}}{\partial E_{q}'} T_{d}^{-1} \left[ E_{q}' - E(h_{10}, E_{q}') - \epsilon \frac{2H_{0}}{2H_{0}} \omega \omega D \omega \right] &= h_{20} + \epsilon h_{21} \\
\epsilon \frac{\partial h_{20}}{\partial E_{q}'} T_{d}^{-1} \left[ E_{q}' - E(h_{10}, E_{q}') - \epsilon \frac{2H_{0}}{2H_{0}} \omega \omega D \omega \right] \\
&= H^{-1} \left[ P_{m} - P(h_{10}, E_{q}') - \epsilon \frac{2H_{0}}{2H_{0}} \omega \omega D \omega \right]
\end{align*}
\]

where matrices \( M_{1} \) and \( M_{4} \) are defined as the Jacobians of functions \( P \) and \( E \) with respect to \( \delta \) calculated along the manifold \( h_{10} \):

\[
M_{1} = \frac{\partial P}{\partial \delta} \bigg|_{\delta = h_{10}} \quad M_{4} = \frac{\partial E}{\partial \delta} \bigg|_{\delta = h_{0}}
\]

Equating the coefficients of \( \epsilon^{0} \):

\[
0 = h_{20} (15) \quad 0 = H^{-1} \left[ P_{m} - P(h_{10}, E_{q}') \right] (16)
\]

Equating the coefficients of \( \epsilon^{1} \):

\[
\begin{align*}
\epsilon \frac{\partial h_{10}}{\partial E_{q}'} T_{d}^{-1} \left[ E_{q}' - E(h_{10}, E_{q}') \right] \\
&= -H^{-1} M_{1} h_{11} (17) \quad 0 = -H^{-1} M_{4} h_{11} (18)
\end{align*}
\]

from which it is clear that

\[
h_{11} = 0
\]

Note that (15) was used to obtain (18). Differentiating (16) with respect to \( E_{q}' \):

\[
M_{1} \frac{\partial h_{10}}{\partial E_{q}'} + M_{2} = 0 (19)
\]
where $M_2$ is defined as:

$$M_2 = \frac{\partial P}{\partial E_q} \bigg|_{\delta-h_{10}}$$

From (19) we can write the Jacobian of the slow manifold $h_{10}$ for the generator angles as a function of the matrices $M_1, M_2$:

$$\frac{\partial h_{10}}{\partial E_q} = -M_1^{-1}M_2$$  \hspace{1cm} (20)

Finally, the $O(\epsilon^2)$ approximation of the integral manifold for the shaft variables is given by:

$$\delta_s = h_{10}$$  \hspace{1cm} (21)

$$\omega_s = -cM_1^{-1}M_2T_d^{-1}[E_f - E(h_{10}, E_q)]$$  \hspace{1cm} (22)

where $h_{10}$ is the implicit function of $E_q'$ defined by the solution of:

$$P(h_{10}, E_q') = P_m$$  \hspace{1cm} (23)

Variables $\delta_s, \omega_s$ are the slow components of the fast variables corresponding to the shaft dynamics and are functions of $E_q'$.

Note that according to (22) the frequency deviations during slow transients are of order $\epsilon$, although the angles may vary considerably to achieve the power balance (23). This is characteristic of the classical description of voltage stability problems, where large voltage variations are accompanied by negligible frequency errors. As the value of the frequency error is small it is reasonable to assume that the frequency control loop is not excited during slow flux-voltage transients.

### 11.2 Slow flux and voltage dynamics: SNB

An approximate decomposed version of the slow flux dynamics is derived from (10) by replacing $\delta$ by $h_{10}$, which is obtained by solving (23). Thus:

$$E_q' = T_d^{-1}[E_f - E(h_{10}, E_q')$$  \hspace{1cm} (24)

The linearized version of (24) around a equilibrium point $E_{eq}'$ gives the following state matrix, obtained using the definitions of the M matrices and (20):

$$A_s = T_d^{-1}M_{3a} + M_{4a}M_1^{-1}M_{2a}$$  \hspace{1cm} (25)

where:

$$M_3 = -\frac{\partial E}{\partial E_q} \bigg|_{\delta-h_{10}}$$

and the subscript ‘o’ denotes evaluation at the equilibrium point $E_{eq}'$.

The matrices $M_1, M_4$ defined in this and the previous section, are direct generalizations of the linearization coefficients $K_1, K_4$ used in the Heffron–Phillips model of a synchronous machine. There exists an exact one to one correspondence, with the exception of $M_3$ which corresponds to $-1/K_3$ of the single machine model.

Note that the approximate state matrix of the slow machine dynamics (25) is the same matrix that was introduced in Reference 15 as a ‘voltage stability matrix’. Following the above analysis, this matrix approximates with an $\epsilon^2$ error the flux dynamics of a multimachine system without AVR. Moreover, a zero eigenvalue of this matrix corresponds exactly to a saddle node bifurcation of the original system, as can be easily verified by linearizing (1)–(3) as in Reference 15. The SNB condition of this system is given by:

$$\det[M_{3b} + M_{4b}M_1^{-1}M_{2b}] = 0$$  \hspace{1cm} (26)

A saddle node bifurcation in a multimachine system will result in a slow flux decay felt by all generators. This will have a similar drifting effect on the generator terminal and load bus voltages leading the whole system either to a voltage collapse, or to the loss of synchronism between the generators.

### 11.3 Fast dynamics: electromechanical oscillations

Once an approximate slow manifold has been found from (21) and (22), the fast dynamics of the multimachine system can be reconstructed using the off-manifold variables defined below:

$$\delta_f = \delta - \delta_s$$  \hspace{1cm} (27)

$$\omega_f = \omega - \omega_s$$  \hspace{1cm} (28)

The off-manifold dynamics are described with an $O(\epsilon^2)$ approximation by the following differential equations, which are derived by differentiating (27) and (28) as in Reference 12 and omitting the $\epsilon^3$ and higher order terms. Note that the slow components $\delta_s, \omega_s$ are functions of $E_q'$ and therefore their time derivatives have to be evaluated using the chain rule, i.e. (21) and (22) are differentiated with respect to $E_q'$ and multiplied by $E_{eq}$ given by (10).

$$\epsilon \delta_f = c\delta_{21} + \omega_f + cM_1^{-1}M_2T_d^{-1}[E_f - E(h_{10} + \delta_f, E_q')]$$  \hspace{1cm} (29)

$$\epsilon \omega_f = H^{-1}[P_m - P(h_{10} + \delta_1, E_q') - \frac{\epsilon}{2H}D\omega_f]$$  \hspace{1cm} (30)

In linearizing equations (29) and (30) around a point lying on the slow manifold, for which $\delta_f = 0$, the dependence of $h_{10}, h_{21}$ on $E_q'$ has to be taken into account by applying the chain rule once more, i.e.

$$\epsilon \delta_f = \frac{\partial h_{21}}{\partial E_q'} \Delta E_q' + \Delta \omega_f$$

$$+ cM_1^{-1}M_2T_d^{-1} \left[ \left( M_3 - M_4 \frac{\partial h_{10}}{\partial E_q'} \right) \Delta E_q' - M_4 \Delta \delta_f \right]$$

Substituting $h_{21}$ from (17) and using (20) it becomes clear that the slow variables $\Delta E_q'$ are eliminated from the above equation. The same is true when linearizing (30).

Now, in order to return to the original variables we define:

$$\omega_f = (1/\epsilon)\omega_f$$  \hspace{1cm} (31)

and the following linearized state equation for the off-manifold, electromechanical oscillation dynamics is obtained:

$$\begin{bmatrix} \Delta \delta_f \\ \Delta \omega_f \end{bmatrix} = \begin{bmatrix} -M_1^{-1}M_2T_d^{-1}M_4 & I_m \\ -T_d^{-1}M_1 & -T_d^{-1}D \end{bmatrix} \begin{bmatrix} \Delta \delta_f \\ \Delta \omega_f \end{bmatrix}$$  \hspace{1cm} (32)

where $I_m$ is the $m \times m$ identity matrix. The $M$ matrices for the fast dynamics state equations (32) are computed at an equilibrium point of the off-manifold dynamics $\delta_f = 0$, i.e. a point lying on the slow manifold. This is not necessarily an equilibrium point of the system, because the slow dynamics may not be at equilibrium at this point.

One interesting aspect of (32) is that it demonstrates mathematically that the field winding is introducing
positive damping torque to the fast electromechanical oscillations through the matrix block $-M_1^{-1}M_2T_2^{-1}M_4$. Therefore, an unregulated system is not expected to demonstrate oscillatory instability, as discussed in greater detail in Reference 16.

II.4 Review of the assumptions
Let us consider now the assumptions implicitly made during this brief presentation.

(1) For non-impedance loads the functions $P$ and $E$ are not unique, because they depend on the solution of the network. Our assumption is that, starting from a normal operating point, the system remains within one causal region. As the algebraic constraints have no singular points inside the causal region, the functions $P$ and $E$ remain unique and they are the ones corresponding to the normal operation of the system.

(2) The decomposition presented here is possible only when the fast dynamics of the system (i.e. the electromechanical oscillations) are stable. It is also clear from Section II.1 that a condition for the existence of a slow manifold is that equation (23) has a solution. Therefore, $M_1$ should be nonsingular. We will return to this point when discussing the first illustrative example.

(3) For any sudden disturbance, the pre-disturbance conditions must belong to the region of attraction of the post-disturbance stable equilibrium of the off-manifold dynamics.

III. System with regulated machines
The introduction of AVRs in the multimachine system (1)-(3) interferes with the time-scale decomposition performed in the previous section for the following reasons.

(1) It is well known that excitation controllers contribute, under certain conditions, negative damping to the electromechanical oscillations. This may result in a violation of the second assumption of Section II.4 that requires the stability of fast shaft dynamics.

(2) High excitation system gains tend to force the machine flux variables $E'_q$ to become fast. Thus, the basis for the decomposition into fast and slow dynamics is destroyed.

As a consequence, the problem of time-scale decomposition when all the generators in a power system are under automatic voltage regulation, has not been addressed in this paper.

Another, perhaps more challenging, problem is the analysis of a power system when some of its generators are regulating and some are not, being either under manual control, or at their excitation limit. This problem was discussed for instance in Reference 18. In this situation there are some slow state variables associated with the unregulated machine flux dynamics, so that a time-scale decomposition is possible, at least in principle, provided that the stability of the electromechanical oscillations is maintained.

At this stage a formal decomposition similar to the one presented in Section II is not available for the partly regulated case. Instead we suggest an alternative, less rigorous formulation, that gives good approximate results under certain circumstances.

This approximate method assumes an algebraic equivalent of the regulated machines similar to the PV bus representation of the classical load flow. Following this the slow dynamics state equations (24) and (25) for the reduced system consisting only of the unregulated machines are formulated. For low excitation gains this process is not accurate because of the AVR voltage droops. For high gain exciters, however, a constant voltage equivalent for the regulated machines can provide a first approximation, as will be seen in the next section.

IV. Illustrative case studies
At this point three case studies are introduced as illustrative examples. The first case is a simple single-machine infinite-bus system used to explain the mechanism of voltage instability and loss of synchronism. In the second case the validity of the time-scale decomposition introduced is demonstrated on a multimachine CIGRE test system. The third case study again involves a single machine and it is selected to represent the salient features of a real world incident documented in Reference 8 as a voltage collapse.

IV.1 Single-machine infinite-bus system
In this example we will re-examine in a new light the classical power-angle curve of a single-machine infinite-bus system. The machine is represented as a one-axis model (1)-(3), having both angle and flux dynamics. To make things as simple as possible we consider a lossless system and a round rotor machine with $X_d = X_q$. The equilibrium curve of this system is given as:

$$P_m = \frac{E_f V}{X_d} \sin \delta$$

(where $V$ is the constant voltage of the infinite bus) and corresponds to the familiar sinusoidal curve drawn in the angle-power plane for constant excitation voltage $E_f$, with the SNB exactly at $\delta = 90^\circ$, as shown in Figure 1.

Equation (23) defining the slow manifold coincides in this case with the ‘transient’ power-angle curve given by:

$$P_m = \frac{E'_q V}{X_d} \sin \delta - \frac{V^2}{2} \left(\frac{1}{X'_d} - \frac{1}{X_q}\right) \sin 2\delta$$

The transient power-angle curves for three different values of $E'_q$, ranging from 0.85 to 1.0, are also shown in Figure 1 with dotted lines.

![Figure 1. Steady state and transient power-angle curves](image-url)
Starting from no load and slowly increasing the machine loading the trajectory of the system will follow the equilibrium curve, along the solid line, up to the SNB point. At this point the machine will not lose synchronism, because the synchronizing coefficient, which is the slope of the transient power-angle curve defined above, is still positive. However, because the excitation level is inadequate to maintain the required power transfer at steady state, the trajectory will depart from the equilibrium curve. The field flux will begin to decrease, and the machine angle will start increasing very slowly, with negligible frequency error according to (22), along the constant power line. If this voltage degradation does not bring about a serious disruption of the system operation, the machine will eventually lose synchronism at the point, where the transient power-angle curve becomes tangent to the constant power line. At this point the synchronizing coefficient \( K_1 \) will become zero, which in the single machine case is equivalent to the singularity of the synchronizing matrix \( M_1 \).

We return now to the question that was left open before closing Section II, i.e. the possibility of a singular \( M_1 \). For a single machine connected to an infinite bus through a lossless line it can be shown that \( M_1 \) becomes singular always at a loading level that is higher than that producing a SNB [3]. Although a similar strict proof is not available for the general case, it is reasonable to expect that (due to the highly reactive nature of the power networks) the SNB of an unregulated multimachine system will precede the loss of synchronism through a singularity of the synchronizing matrix, as shown in Figure 1.

### IV.2 Multimachine test system

As mentioned above, the system studied here is taken from a CIGRÉ task force [5]. The same system has been studied extensively in Reference 7 for various loading scenarios. A single line diagram is shown in Figure 2. The operating point studied here is one for which machine M2 has tripped. Therefore, this is a five-machine system with two infinite buses, shown in the lower part of Figure 2.

Two cases are analysed for this system. Suppose first that all five machines are without AVR. The eigenvalues of the full model of the system are shown in Table 1, together with those of the reduced fast and slow subsystems described by equations (32) and (25), respectively. The \( \varepsilon \) accuracy obtained by the time-scale decomposition applied is self-evident. Note how the time-scale decomposition is possible even after the SNB of the unregulated system. In fact, only the stability of the electromechanical dynamics is required.

Let us look now at the same system when some of the machines are regulating: In the simulated scenario, machines M1, M3 and M4 have reached their overexcitation limits and therefore they have lost voltage control. This leaves three unregulated machines connected to the rest of the system. In a first attempt to model this situation, the dynamics of the regulated machines are ignored, as was discussed in Section III, and the slow dynamics of the remaining machines are formed. The eigenvalues of the full system and those of the three-machines slow subsystem are shown in the first two columns of Table 2.

As one real eigenvalue is very small, the system is close to a SNB, which is due to the field limitation of the three machines. This situation is reflected in the simplified three-machine formulation, although the actual value of the critical eigenvalue is not predicted accurately. As discussed in Section III, this is due to the small value of the excitation gains of the two remaining controlled generators: Machine M5 has an excitation gain of 15 p.u. and a time constant of 0.3 s, and machine M6 a gain of 35 p.u. and a time constant of 0.1 s. By increasing the gains to 95 p.u. and 135 p.u., respectively, and reducing the time constant of M5 to 0.1 s, the eigenvalues of the third column of Table 2 are obtained. Clearly in this case the three machine simplified equivalent which is still the one shown in the second column of Table 2 gives quite accurate results.

The following conclusions can be drawn from this experience: The slow dynamics matrix (25) predicts accurately a SNB of a multimachine system with no AVRs. It

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**Table 1. Eigenvalues without AVR**

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>Fast subsystem</th>
<th>Slow subsystem</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.240 \pm j 6.81)</td>
<td>(-0.241 \pm j 6.82)</td>
<td>(-0.243 \pm j 6.82)</td>
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<tr>
<td>(-0.280 \pm j 6.59)</td>
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<td>(-0.250 \pm j 6.16)</td>
<td>(-0.250 \pm j 6.16)</td>
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<tr>
<td>(-0.189 \pm j 3.98)</td>
<td>(-0.190 \pm j 3.98)</td>
<td>(-0.190 \pm j 3.98)</td>
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<td>(+0.283)</td>
<td>+0.285</td>
<td>+0.285</td>
<td></td>
</tr>
<tr>
<td>(-0.302)</td>
<td>(-0.302)</td>
<td>(-0.302)</td>
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</tr>
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<td>(-0.226)</td>
<td>(-0.226)</td>
<td></td>
</tr>
<tr>
<td>(-0.276)</td>
<td>(-0.276)</td>
<td>(-0.276)</td>
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</tr>
<tr>
<td>(-0.199)</td>
<td>(-0.199)</td>
<td>(-0.199)</td>
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</tr>
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**Table 2. Eigenvalues with AVRs on machines M5 and M6**

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th>Three mach. appr.</th>
<th>Full model</th>
<th>Slow gains</th>
<th>High gains</th>
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<tr>
<td>(-0.325 \pm j 3.90)</td>
<td>(-0.326 \pm j 3.90)</td>
<td>(-0.326 \pm j 3.90)</td>
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<tr>
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<tr>
<td>(-0.243 \pm j 6.16)</td>
<td>(-0.266 \pm j 6.13)</td>
<td>(-0.266 \pm j 6.13)</td>
<td></td>
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<tr>
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<td>(-0.264 \pm j 6.60)</td>
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<tr>
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<td>(-0.251 \pm j 6.75)</td>
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<td>(-5.10 \pm j 4.84)</td>
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</tbody>
</table>
IV.3 A realistic SNB

The system in this third case study is a single-machine system inspired from a B.C. Hydro, North Coast Region incident of 1979 reported in Reference 8. The parameters chosen for the test system are not those of the actual system, so that the similarity is limited only to the structure of the system (an isolated generating plant feeding a large load, weakly connected to the rest of the system), the instability scenario (partial load tripping in an aluminium plant), and the loading levels. This case study is meant to demonstrate the following points.

- An SNB of a synchronous generator can be felt as a voltage instability.
- A voltage instability is possible even with constant impedance loads (whereas an algebraic singularity is impossible in this case).
- When a disturbance takes the operating point very close to an SNB it takes minutes for the instability to evolve.

The single line diagram is shown in Figure 3. The test system consists of an equivalent generator under manual excitation control, a load bus feeding a constant impedance, unity power factor load, and an infinite bus connected to the load bus through a long tie-line. The generator is modelled with one-axis rotor flux dynamics, as in Section II.1. The parameters of the system and the initial conditions are shown in Table 3 on a 1000 MVA basis.

The equilibrium P–V curve of the load bus is drawn in Figure 4 for constant generator output and excitation voltage \( E_f \), because the machine is on manual voltage control. This curve is not similar to the traditional 'nose curve'. The main difference is that there are two possible SNB points for a system operating initially at point A, one for high load (point C) and one for light load (point SNB). The light load bifurcation point is brought about by the power transfer limit on the long tie line. Note that this SNB point is in the upper part of the P–V curve, meaning that the security margin would be larger for a constant power than for a constant impedance load.

A sudden load admittance reduction at time \( t = 0 \), from \( G_A = 0.63 \) p.u. (corresponding to point A) to \( G_B = 0.5 \) p.u. (corresponding to the almost tangent load characteristic close to the SNB point in Figure 4) is simulated next. The results are shown in Figure 5. The following points are worth noting.

- All responses have a fast transient part corresponding to the electromechanical oscillation mode, which dies out in a few seconds after the disturbance.
- The effect of the SNB is evident both in the machine demagnetization and in the rotor angle upward drift. Synchronism is eventually lost 142 seconds after the disturbance.
- During the two minute interval prior to the loss of synchronism there are no observable effects either in frequency, or in the generated power of the synchronous machine.
- As soon as synchronism is lost, the generator power output drops abruptly and the frequency rises rapidly.
- Before the loss of synchronism, the apparent power flow on the tie line has reached 650 MVA, becoming double the value immediately after the disturbance, and the load bus voltage has dropped close to 0.7 p.u.

It is interesting to compare the above remarks on the simulated test case of Figure 5 with the following comments on the actual B.C. Hydro incident taken from Reference 8.

‘Although constant impedance load characteristic is considered soft and least aggravating in voltage stability studies, this case shows the fallacy of such generalizations... The output of generators was fairly steady within the observed duration, which excludes the possibility of angle instability... The system separated when (i) the current at one end exceeded the overcurrent relay setting; (ii) the apparent impedance at the other end was within the out-of-step relay setting.’

V. Conclusions

This paper has shown how models containing both ‘angle’ dynamics and ‘voltage’ dynamics can be decomposed into
When some machines have active voltage regulators, the reduced-order slow ‘voltage’ model consisting of only the field flux dynamics of excitation-limited machines can serve as an approximation of the unregulated subsystem slow dynamics. This approximation gives good results only for high excitation gains of the regulating machines. Further research is necessary in order to achieve a more accurate model by using a formal time-scale decomposition of the partly regulated system.

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VII. References

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